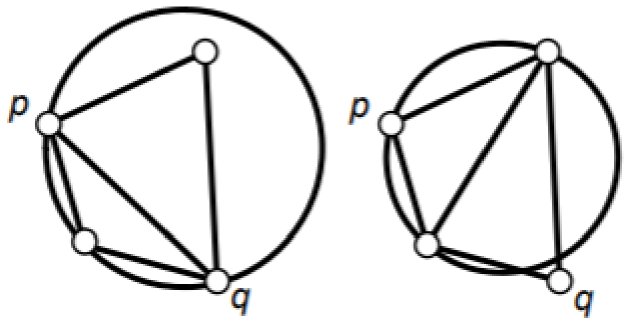
Triangulations and Related Problems

Delaunay Triangulation

- Reminder – a triangulation which maximize the minimum angle in the triangulation.
- A triangle is Delaunay iff the circle through its vertices is empty of other sites.

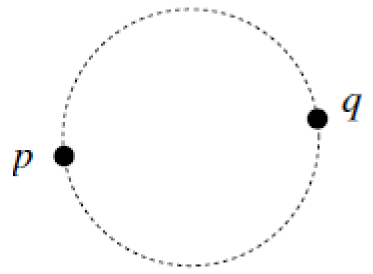


Other kinds of graphs

- Euclidean minimum spanning tree ($EMST$) – A set of edges spanning the a set of points with the minimum total edge length.
- Relative neighborhood graph (RNG) – An edge (p, q) is a part of the RNG iff

$$d(p,q) \leq \min_{r \in P, r \neq p,q} \max(d(p,r), d(r,q))$$

- Gabriel Graph (GG) - Two points p and q are connected by an edge of the GG if and only if the disc with diameter pq does not contain any other point of *P*.



Other kinds of graphs

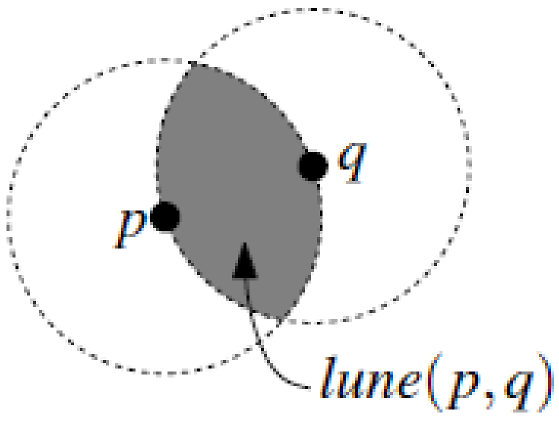
- Prove that
$$EMST \subseteq RNG \subseteq GG \subseteq DT$$
- The last relation is part of the HW, we will show the other two.
- We will start by understanding the RNG better.

Relative neighborhood graph

- Relative neighborhood graph (RNG) – An edge (p, q) is a part of the RNG iff

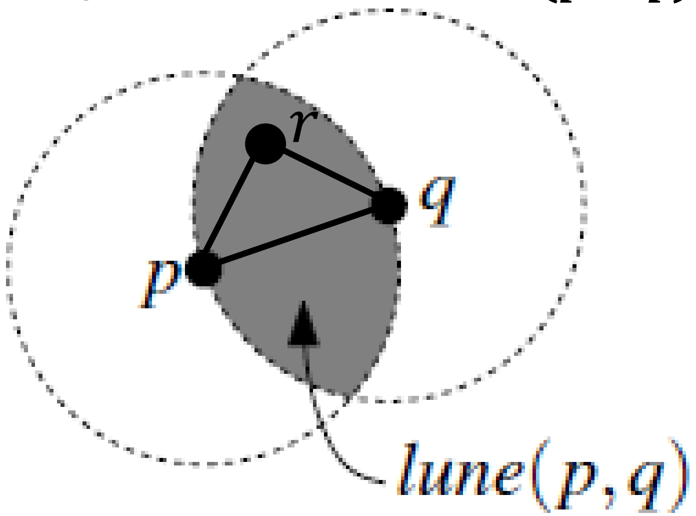
$$d(p,q) \leq \min_{r \in P, r \neq p,q} \max(d(p,r), d(r,q))$$

- <u>Claim:</u> The edge (p, q) is part of the RNG iff the *lune of* p and q is empty.
- It is easy to see from the definition.



$EMST \subseteq RNG$

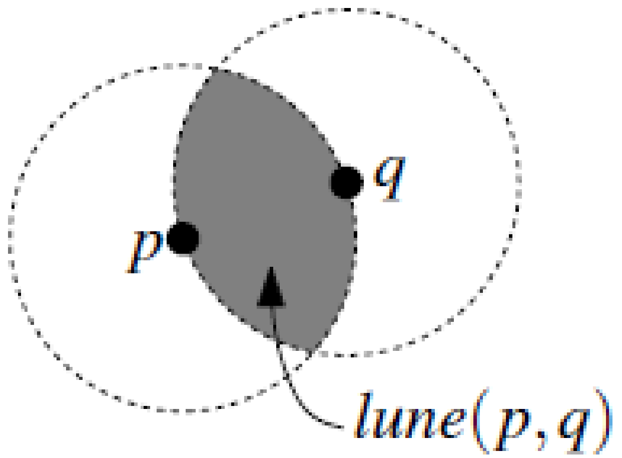
- Let $(p, q) \in EMST(P)$, and assume that $(p, q) \notin RNG(P)$
- That means that there exist point $r \in lune(p, q)$

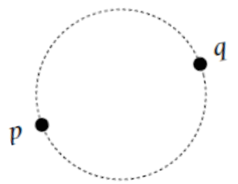


- The edge (p, q) is the largest in the circle pqr, and thus, can not be part of $EMST(P)$.
  - Recall Algo 1 MST rules

$RNG \subseteq GG$

- Reminder: $(p, q) \in RNG(P)$ iff $lune(p, q)$ is empty.
- $(p, q) \in GG(P)$ iff the the disc with diameter pq does not contain any other point of P.





- The circle is subset of the lune, and thus the claim.

Euclidean Traveling Salesman Problem

- The *traveling salesman problem* (TSP) is to compute a shortest tour visiting all points in a given point set.
- The traveling salesman problem is NP-hard.
- In the Euclidean version the distances are the Euclidean distance.
- Show how to find a tour whose length is at most two times the optimal length.

Euclidean Traveling Salesman Problem

- Claim: The optimal tour length is longer (or equal) to the $EMST$ weight
- Proof: Consider the graph created by the TSP tour, this graph spans the set of points and thus its weight is greater than the $EMST$ weight.
- Claim: The length of a DFS traversal over the $EMST$ is at most twice the length of the $EMST$ (and thus, at most twice the length of the optimal tour).
- Proof: Each edge is traversed at most twice.
- TSP Approximation algorithm: Find the $EMST$, and return a DFS traversal tour.
- Complexity?

Computing the EMST

- What is the best algorithm to compute an $EMST$?
- Using Prim's/Kruskal's algorithm the complexity will be $O(n^2)$.
  - Why?
- Can we do better?
- Recall that $EMST \subseteq DT$
- Compute the DT of the set of points - $O(n \log n)$
- Compute the $EMST$ of the DT using Prim's/Kruskal's algorithm - $O(n \log n)$
- Total complexity - $O(n \log n)$